TD

# Exercice 1

(S1)  
x + y + 2z = 5 => x + y + 2z = 5 => x + y + 2z = 5 => x + y + 2z = 5  
x – y – z = 1 -2y – 3z = 1 – 5 = -4 (E2) – (E1) y + 3/2z = 2 (E2)/(-2) y + 3/2z = 2  
x + z = 3 -y – z = -2 (E3) – (E1) -y – z = -2 1/2z = 0 (E3) + (E2)

x = 5 – 2 – 2 \* 0 = 3  
y = 2 – 3/2 \* 0 = 2  
z = 0  
S = {(3, 2, 0)}

(S2)  
2x – 3y + 6z + 2t = 5 => 2x – 3y + 6z = 5 – 2t  
y – 2z + t = 1 y – 2z = 1 - t  
z – 3t = 2 z = 2 + 3t

x = 1/2(5 – 2t + 3y – 6z) = 1/2(5 – 2t + 3(5 + 5t) -6(2 + 3t)) => x = 4 – 5/2t  
y = 1 – t + 2(2 + 3t) = 5 + 5t y = 5 + 5t  
z = 2 + 3t z = 2 + 3t  
S = {(4 – 5/2t, 5 + 5t, 2 + 3t, t), t € R}

(S4)  
2x + y – 2z = 10 <=> -2z + y + 2x = 10 => -2z + y + 2x = 10  
2x + 2y + 2t = 1 2y + 2t + 3x = 1 2y + 2t + 3x = 1  
5x + 4y + z + 3t = 14 z + 4y + 3t + 5x = 14 9/2y + 3t + 6x = 19 (E3) + (E1)/2

-2z + y + 2x = 10 => -2z + y + 2x = 10 => 2z + y + 2x = 10  
2y + 2t + 3x = 1 2y + 2t + 3x = 1 2y + 2t + 3x = 1  
(3 + 9/4 \* 2)t + (6 – 9/4 \* 3)x = 19 – 9/4 -3/2t – 3/4x = 67/4 t = -2/3(67/4 + 3/4x) = -67/6 – 1/2x

-2z = 10 – 2x – (35/3 – x) = -5/3 – x => z = 5/6 + 1/2x  
2y = 1 – 3x – 2(-67/6 – 1/2x) = 70/3 – 2x y = 35/3 - x  
t = -2/3(67/4 + 3/4x) = -67/6 – 1/2x t = -67/6 – x/2  
S = {(x, 35/3 – x, 5/6 + x/2, -67/6 – x/2), x € R}

(S7)  
x – y + 2z = 0 => x = y – 2z  
S = {(y – 2z, y, z), (y, z) € R2}

(S8)  
S = {(1/5z + 1/5t, -3/5z – 1/5t, z, t), (z, t) € R2}

# Exercice 2

(S2)  
x + iy + 2z = 0 => y = -3z + 2iz  
ix + 3z = 0 x = 3iz  
S = {z(3i, -3 + 2i, 1), z € C}

(S5)  
x + y + z = 1 => x + y + z = 1 …………………  
2x + iy – z = I (-2 + i)y – 3z = -2 + i  
-ix + (1 + i)y + 2iz = 1 (1 + 2)y + 3iz = 1 + i  
S = O/

# Exercice 4

(S2)  
3x – y – 2z = u => x – 3y + z = t => x – 3y + z = t => x – 3y + z = t  
-x + 3y – z = v -x + 3y – z = v 0 = t + v -8y + 5z = w + 2t  
-2x – 2y +3z = w -2x – 2y +3z = w -8y + 5z = w + 2t 0 = u – t + w <=> u = t - w  
x – 3y + z = t 3x – y – 2z = u 8y – 5z = u – 3t 0 = t + v

(S2) a au moins une solution si et seulement si u – t + w = 0 => u – t + w = 0 => u = -w - v  
 t + v = 0 t = -v t = -v

x – 3y + z = -v => x – 3y + z = -v => x – 3y + z = -v => x = 3y – z + v  
-8y + 5z = w – 2v -8y = -5z + w – 2v y = 1/8(5z – w + 2v) y = 1/8(5z – w + 2v)

x = 7/8z – 3/8w – 1/4v  
y = 1/8(5z – w + 2v)  
S2 = {(7/8z – 3/8w – 1/4v, 1/8(5z – w + 2v), z), z € R et u = -w – v et t = -v, (v, w) € R2}

(S1)  
x + 3y + 6z = u => x + 3y + 6z = u => x + 3y + 6z = u => x + 3y + 6z = u  
3x + y + 3z = v 3x + y + 3z = v -8y – 15z = v – 3u -8y – 15z = v – 3u  
6x + 6y + z = w 6x + 6y + z = w -12y – 35z = w – 6u -25z = 2w – 3u – 3v  
7x + 9y + 7z = t 0 = t – u – w <- -L1 – L3  0 = t – u – w 0 = t – u – w

(S1) a une solution ssi t = u + w et donc

x = -3y – 6z + u => x = -3y – 6z + u  
y = -1/8(15z + v -3u) y = -15/8(-2/25w + 3/25v + 3/25u) – 1/8v + 3/8u  
z = -2/25w + 3/25u + 3/25v z = -2/25w + 3/25v + 3/25u

x = -17/100u + 33/100v + 3/100 w  
y = 3/20w – 7/20v + 3/20u  
z = -2/25w + 3/25v + 3/25 u

Sous la condition t = u + w avec (u, v, w) € R3 on a S = {(x, y, z)}

# Exercice 3

(S1)  
x + y + z + t = 0 => x + y + z + t = 0 => x + y + z + t = 0 => x + y + z + t = 0 => x = -y -z -t  
2x – y – z + 3t = 0 -3y – 3z + t = 0 -3y – 3z + t = 0 -3y – 3z + t = 0 -3y = 3/4 + 5/4t  
x – 2y + 2z – t = 1 -3y + z – 2t = 1 4z – 3t = 1 4z – 3t = 1 z = 1/4 + 3/4t  
2x + 2y – 2z + 5t = -1 -4z + 3t = -1 -4z + 3t = -1 0 = 0

Oui ptdr

(S3)  
x1 + x3 + x5 + x6  
x1 + x6  
x2 + x4 + 2x6  
x1 + x2 + x3 + x4 + x5 + 3x6  
S = {x5(0, -1, -1, 1, 1, 0) + x6(-1, -1, 0, -1, 0, 1), (x5, x6) € R2}

# Exercice 5

1. v-> = 3e1-> + 4e2->
2. v-> = e1 + e2 – e3
3. ?
4. v-> = 0-> (= 0e1 + 0e2 + 0e3)

# Exercice 6

* E =/ Ø (0 + 0 + 0 = 0 =/ Ø)
* Soient u->(x1, y1, z1) € E d’où x1 + y1 + z1 = 0  
    
  (x1, y1, z1) + () = (x1 + x2, y1 + y2, z1 + z2)

# Exercice 10

b-> = (0, -1, 2, 0)

f1-> + f2-> = 2b->  
5f1-> + f3-> = 3b-> (\*)  
4g1-> + g2-> = 3b->

F c G <=> (f1-> € G et f2-> € G et f3-> € G)

Avec (\*) en déduire que  
F = Vect(f1->, b->)  
G = Vect(g1->, b->)

1. Mq F = Vect(f1->, b->)  
   Vect(f1->, b->) c F car f1-> € F et grâce à (\*), on a b-> = (1/2)(f1-> + f2->) € F   
   F c Vect(f1->, b->) car f1-> € Vect(f1->, b->) et on a f2-> = -f1-> + 2b-> et f3-> = -5f1-> + 3b->
2. Mq G = Vect(g1->, b->)  
   Vect(g1->, b->) c G car g1-> € G et on a b-> = 4/3g1-> + 1/3g2-> € G  
   G c Vect(g1->, b->) car g1-> € Vect(g1->, b->) et on a g2-> = 3b-> - 4g1->
3. Mq F = G  
   F c G ?  
   Existe-t-il (α, β) € R² tq f1-> = αg1-> + βb-> ?   
   f1-> = (-α, - α, α, - α) + (0, - β, 2 β, 0)  
   - α = 1 => α = 1  
   - α – β = 0 β = 1  
   α + 2β = 1  
   - α = 1  
     
   g1-> = -f1 + b  
   donc g1-> € Vect(f1->, b->) = F et b-> € F donc Vect(g1->, b->) c F  
     
   F c G et G c F donc F = G

# Exercice 13

1. E n F c E c E + F  
   E n F c F c E + F

f-> € E n F <=> f-> € E et f-> € F d’où E n F c E (1) et E n F c F (3)

Soit e-> € E  
e-> = e-> + 0->

F espace vectoriel donc 0-> € F d’où E c E + F  
De même F c E + F

1. E = {v-> = (v1, v2, v3, v4) | v1 + v2 + v3 + v4 = 0 et v2 + 2v3 + 3v4 = 0}  
   F = Vect((1, 2, 0, 0), (1, -1, -1, 1))  
   1. On cherche a-> € E et a-> €/ F  
      F = {(α + β, 2α – β, -β, β), (α, β) € R²}  
        
      v-> = (v1, v2, v3, v4)  
      v3 = v4 = 1 => v-> €/ F  
        
      v-> € E <=> v1 + v2 + 1 + 1 = 0 et v2 + 2 + 3 + 0  
       v1 = 3 et v2 = -5  
        
      v-> €/ E n F
   2. ??????????????????
   3. b-> € F et b-> €/ E n F ?  
      b-> € F et b-> €/ E   
      b-> = (1, 2, 0, 0) convient b-> € F  
      1 + 2 + 0 + 0 =/ 0 donc b-> €/ E

# Exercice 20

e1-> = (1, -1, 1)  
e2-> = (2, -1, 3)  
e3-> = (-1, 1, -1)

(e1->, e2->, e3->) n’est pas libre

On voit que e3-> = -e1-> donc 1 \* e1-> + 0 \* e2-> + e3-> = 0->

On a d1e1-> + d2e2-> + d3e3-> = 0-> avec (d1, d2, d3) =/ (0, 0, 0)

La famille n’est pas libre

Remarque :

(e1->, e3->) pas libre => (e1->, e2->, e3->) pas libre

# Exercice 21

1. (u->, v->) = ((1, 1, 0), (4, 1, 4))  
   d1u-> + d2v-> = 0->  
   0 = d1 + 4d2 => 0 = 0  
   0 = d1 + d2 0 = d1  
   0 = 4d2 0 = d2  
   d1 = d2 = 0 => famille libre  
     
   (u->, w->) = ((1, 1, 0), (2, -1, 4))  
   d1u-> + d2w-> = 0->  
   0 = d1 + d2 => 0 = 0  
   0 = d1 – d2  0 = d1  
   0 = 4d2 0 = d2  
   d1 = d2 = 0 => famille libre  
     
   (v->, w->) = ((4, 1, 4), (2, -1, 4))  
   0 = 4d1 + 2d2 => 0 = d1  
   0 = d1 – d2  0 = d2  
   0 = 4d1 + 4d2 -d1 = d2  
   d1 = d2 = 0 => famille libre
2. (u->, v->, w->) = ((1, 1, 0), (4, 1, 4), (2, -1, 4))  
   0 = d1 + 4d2 + 2d3 => 0 = d3 => 0 = d3  
   0 = d1 + d2 – d3 -d1 = -2d3  0 = d1  
   0 = 4d2 + 4d3  -d2 = d3  0 = d2  
   d1 = d2 = d3 = 0 => famille libre OK full drunk en fait c’est pas libre

# Exercice 22

1. A = ((2, 3, 0), (0, 1, 4))  
   0 = d1 => 0 = d1  
   0 = 3d1 + d2 0 = 0  
   0 = 4d2 0 = d2  
   Famille libre  
     
   E = Vect(e->, f->)  
   (e->, f->) est génératrice de Vect(e->, f->) donc c’est une base du Vect(e->, f->)
2. B = ((0, 1, -1), (1, 2, -1), (0, 2, 1), (4, 6, 3))  
   0 = d2 + 4d4 =>   
   0 = d1 + 2d2 + 3d3 + 6d4  
   0 = -d1 – d2 + d3 + 3d4

# Exercice 26

u-> = (0, 1, 1)  
v-> = (1, 1, 0)  
w-> = (1, 1, 0)

Base de R3 ?

Coordonnées de t-> = (1, 1, 1) dans cette base ?

(u->, v->, w->) est une base si la famille est libre et génératrice

dim(R3) = 3  
(u->, v->, w->) famille de 3 vecteurs => pour que cette famille soit une base il suffit qu’elle soit libre

on cherche a, b, c tel que au-> + bv-> + cw-> = 0

b + c = 0 => b = 0   
a + c = 0 c = 0  
a + b = 0 a = 0

On a donc une famille libre de 3 vecteurs dans R3 de dimension 3 => base de R3

On cherche les a, b, c tels que au-> + bv-> + cw-> = t->

b = 1/2  
c = 1/2  
a = 1/2

donc t-> = 1/2u-> + 1/2v-> + 1/2w->

dans (u->, v->, w->), t-> a pour coordonnées (1/2, 1/2, 1/2)

# Exercice 24

1. u-> = (1, 2, -1, 0)  
   v-> = (0, 1, -4, 1)  
   w-> = (2, 5, -6, 1)  
     
   a + 2c = 0 => a = -2  
   2a + b + 5c = 0 b = -c  
   -a – b -6c = 0   
   b + c = 0   
   Famille liée donc ne peut pas être complétée en une base
2. u-> = (1, 0, 2, 3)  
   v-> = (0, 1, 2, 3)  
   w-> = (1, 2, 0, 3  
     
   a + c = 0 => a + c = 0 => a + c = 0 => a = 0  
   b + 2c = 0 b + 2c = 0 b + 2c = 0 b = 0  
   2a + 2b = 0 2b – 2c = 0 -4c = 0 c = 0  
   3a + 3b + 3c = 0 3b = 0 -6c = 0  
   Famille libre donc on peut compléter en une base  
   dim(R4) = 4  
   Les bases de R4 ont 4 vecteurs  
   On cherche t-> tel que (u->, v->, w->, t->) soit libre (donc une base)  
   (u->, v->, w->, t->) famille libre de 4 vecteurs de R4 donc base  
   t-> = (1, 0, 0, 0)
3. u-> = (1, -1, 1, -1)  
   v-> = (1, 1, 1, 1)  
     
   a + b = 0 => a + b = 0 => a = 0  
   -a + b = 0 2b = 0 b = 0  
   a + b = 0 0 = 0  
   -a + b = 0 2b = 0

# Exercice 32

R4  
u1-> = (1, 0, 1, -1)  
u2-> = (-1, 1, 1, 2)  
u3-> = (1, -2, -3, -3)  
v1-> = (1, 0, 2, −1)  
v2-> = (0, 1, 0, 3)

1. a – b + c = 0 => a – b + c = 0 => a – b + c = 0 => a = c  
   b – 2c = 0 b – 2c = 0 b – 2c = 0 b = 2c  
   a + b – 3c = 0 2b – 4c = 0 0 = 0  
   -a + 2b – 3c = 0 b – 2c = 0 0 = 0

# Exercice 51

A = ( a b )  
 c d

inversible <=> ad – bc =/ 0

A-1 = 1/(ad – bc)( d -b )  
 -c a

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D = Matrice « diagonale »

D-1 = Diag(1/d1, … , 1/dn)

Diag(d1, … , dn) \* Diag(a1, … , an) = Diag(a1d1, … , andn)

# Exercice 48

1. A :  
   1 1 2 1 0 0   
   1 2 1 0 1 0  
   2 1 1 0 0 1  
     
   1 1 2 1 0 0  
   0 1 -1 -1 1 0 L2 <- L2 - L1  
   2 1 1 0 0 1  
     
   1 1 2 1 0 0  
   0 1 -1 -1 1 0  
   0 -1 -3 -2 0 1 L3 <- L3 – 2L1  
     
   1 1 2 1 0 0  
   0 1 -1 -1 1 0  
   0 0 -4 -3 1 1 L3 <- L3 + L2  
     
   …………………..  
     
   A-1 = 1/4(iqgduyqgcukq)  
     
   Methode 2 : systeme  
   A(x, y, z) = (a, b, c)  
   <=> (x, y, z) = A-1(a, b, c)  
     
   x + y + 2z = a  
   x + 2y + z = b  
   2x + y + z = c  
     
   x + y + 2z = a  
   y – z = a – b  
   -y – 3z = c – 2a  
     
   x + y + 2z = a  
   y – z = a – b  
   -4z = c – a – b  
     
   x = a – 5a/4 + 3b/4 + c/4 + 2c/4 – 2a/4 – 2b/4 = -3a/4 + b/4 + 3c/4  
   y = a – b – c/4 + a/4 + b/4 = 5a/4 -3b/4 - c/4  
   z = -c/4 + a/4 + b/4  
     
   x = -a/4 + b/4 + 3c/4  
   y = -a/4 -3b/4 – c/4  
   z = 3a/4 + b/4 – c/4  
     
   (x, y, z) = D (a, b, c)